Register No : ________________

# THE KAVERY ENGINEERING COLLEGE

*(An Autonomous Institution)*

B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, NOV /DEC 2024

*Third Semester*

*Electronics and Communication Engineering*

MA3355 - Random Processes and Linear Algebra

(Common to Fourth Semester Biomedical Engineering)

Use of Statistical table can be Permitted

*Regulations - 2021*

*Duration : 3 Hrs* *Maximum : 100 Marks*

*PART – A (ANSWER ALL QUESTIONS) (Marks 10*2=20)*

| *Q.No* | *Questions* | *BLT* | *CO* | *Marks* |
|---|---|---|---|---|
| 1. | Write the Mean and Variance of Poisson distribution. | RE | 1 | 2 |
| 2. | If X is uniformly distributed over (0,10), find P(X > 8). | AP | 1 | 2 |
| 3. | Define Covariance. | RE | 2 | 2 |
| 4. | State Central limit theorem. | RE | 2 | 2 |
| 5. | Mention the classification of random processes. | RE | 3 | 2 |
| 6. | Write any two properties of Poisson processes. | RE | 3 | 2 |
| 7. | Determine whether set of all vectors of the form $(a, 0,0)$ is a subspace of $R^3$. | UN | 4 | 2 |
| 8. | Check whether the vectors $v_1 = \{1,-2,3\}$ , $v_2 = \{5,6,-1\}$ and $v_3 = \{3,2,1\}$ form a linearly dependent set. | UN | 4 | 2 |
| 9. | Let $T: R^2 \to R^2$ be a linear transformation that maps $u$ to $T(u) = (3,4)$ and maps $v$ to $T(v) = (-2,5)$. Find $T(2u + 3v)$. | UN | 5 | 2 |
| 10. | Show that $\langle u, v\rangle = 9u_1v_1 + 4u_2v_2$ is the inner product on $R^2$ generated by $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. | UN | 5 | 2 |

*PART – B (ANSWER ALL QUESTIONS) (Marks 5*16 = 80)*

| Q.No | | | Questions | BLT | CO | Marks |
|---|---|---|---|---|---|---|
| 11. | a | i | In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C. | AP | 1 | 8 |
| | | ii | A random variable X has the following probability function. | UN | 1 | 8 |

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| P(x) | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

(i) Determine the value of 'a'.
(ii) Find P(X < 3), $P(X \geq 3)$, $P(0 < X < 5)$
(iii) Find the distribution function of X

Or

| | | | | | | |
|---|---|---|---|---|---|---|
| | b | i | In a large consignment of electric bulbs 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (i) all are good bulbs (ii) atmost there are 3 defective bulbs and (iii) exactly there are 3 defective bulbs. | AP | 1 | 8 |
| | | ii | The weekly wages of 1000 workmen are normally distributed around a mean of Rs.70 with a S.D. of Rs.5. Estimate the number of workers whose weekly wages will be (i) between Rs.69 and Rs.72 (ii) less than Rs.69 and (iii) More than Rs.72. | UN | 1 | 8 |
| 12. | a | i | The joint probability mass function of (x,y) is given by $p(x,y)=k(2x+3y)$, $x=0,1,2$, $y=1,2,3$. Find i)The value of k ii) Marginal distribution of x and y iii) $P(X+Y>3)$. | AP | 2 | 8 |
| | | ii | The joint probability density function of X and Y is given by $f(x,y)=e^{-(x+y)}, x>0, y>0$ find the probability density function of $U=\frac{X+Y}{2}$. | AP | 2 | 8 |

Or

| | | | | | | |
|---|---|---|---|---|---|---|
| | b | | From the following data, find<br>(i) The two regression equations.<br>(ii) The coefficient of correlation between the marks in Economics and Statistics.<br>(iii) The most likely marks in statistics, when marks in Economics are 30. | AP | 2 | 16 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks in Economics | 25 | 28 | 35 | 32 | 31 | 36 | 29 | 38 | 34 | 32 |
| Marks in Statistics | 43 | 46 | 49 | 41 | 36 | 32 | 31 | 30 | 33 | 39 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 13. | a | | Show that the random process $x(t) = A\cos(\omega t + \theta)$ is wide sense stationary, if A and $\omega$ are constants and $\theta$ is uniformly distributed random variable in $(-\pi,\pi)$. | AP | 3 | 16 |

Or

| | | | | | | |
|---|---|---|---|---|---|---|
| | b | | The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3 \ldots$ having 3 states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$. Find (i) $P\{X_2 = 3\}$ and (ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$. | AP | 3 | 16 |
| 14. | a | | Prove that the set $V = R^n$ with standard operations of addition and scalar multiplication is a vector space. | UN | 4 | 16 |
| | | | Or | | | |
| | b | i | Check whether the vector $W = (1,-2,2)$ is a linear combination of vectors in the set $S = \{(1,2,3), (0,1,2), (-1,0,1)\}$. | UN | 4 | 8 |
| | | ii | Find the basis and dimension of the subspace W of $R^4$ spanned by the vectors (1,-2,5,-3) ,(2,3,1,-4) and (3,8,-3,-5). | UN | 4 | 8 |
| 15. | a | i | Let $V$ and $W$ be any two vector spaces. Prove that the mapping $T: V \rightarrow W$ such that $T(v) = 0$ for all $v \in V$ is a linear combination. | AP | 5 | 8 |
| | | ii | Let $T: R^2 \rightarrow R^3$ be defined by $T(x, y) = (x + 3y, 0, 2x - 4y)$. Compute the matrix of the transformation with respect to the standard bases of $R^2$ and $R^3$. Find $N(T)$ and $R(T)$. | AP | 5 | 8 |
| | | | Or | | | |
| | b | | Apply the Gram-Schmidt Orthonormalization process to construct an Orthonormal basis for the subspace $W$ of the Euclidean space $R^3$ spanned by $B = \{(1,1,0), (1,2,0), (0,1,2)\}$. | AP | 5 | 16 |